

*Andrea Vattani*

*k*-means Requires Exponentially Many Iterations Even in the Plane

The *k*-means algorithm is a well-known method for partitioning  $n$  points that lie in the  $d$ -dimensional space into  $k$  clusters. Its main features are simplicity and speed in practice. Theoretically, however, the best known upper bound on its running time (i.e.  $O(n^{kd})$ ) is, in general, exponential in the number of points (when  $kd = \Omega(n / \log n)$ ). Recently, Arthur and Vassilvitskii [3] showed a super-polynomial worst-case analysis, improving the best known lower bound from  $\Omega(n)$  to  $2^{\Omega(\sqrt{n})}$  with a construction in  $d = \Omega(\sqrt{n})$  dimensions. In [3] they also conjectured the existence of superpolynomial lower bounds for any  $d \geq 2$ . Our contribution is twofold: we prove this conjecture and we improve the lower bound, by presenting a simple construction in the plane that leads to the exponential lower bound  $2^{\Omega(n)}$ .